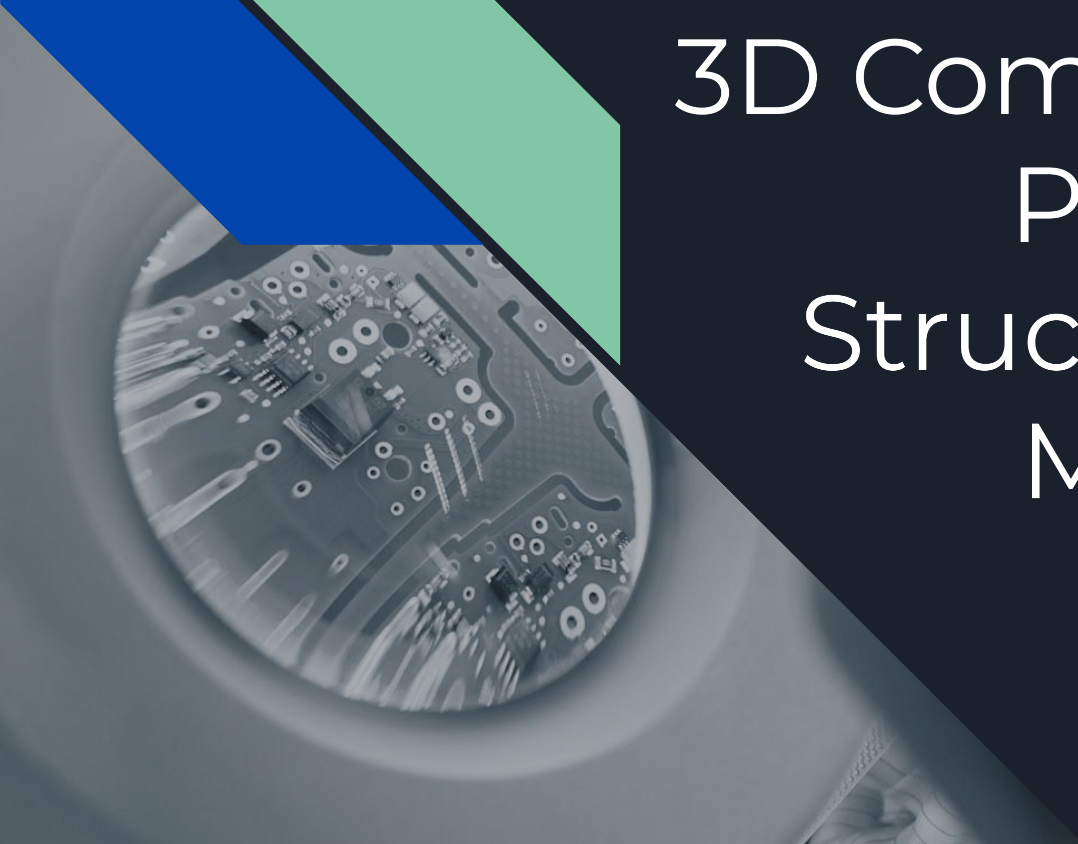

3D Computer Vision Project: Structure From Motion

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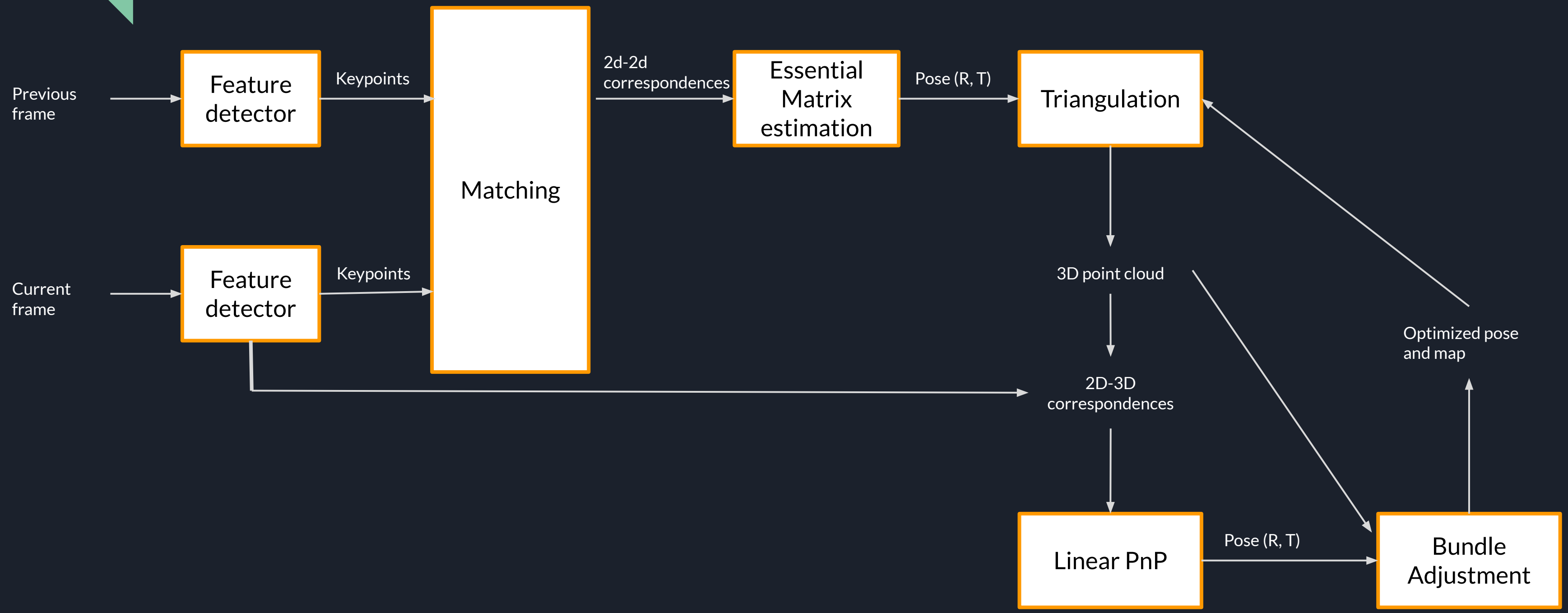
HEVRA PETEKKAYA (7055462)
HEWAN SHRESTHA (7047533)

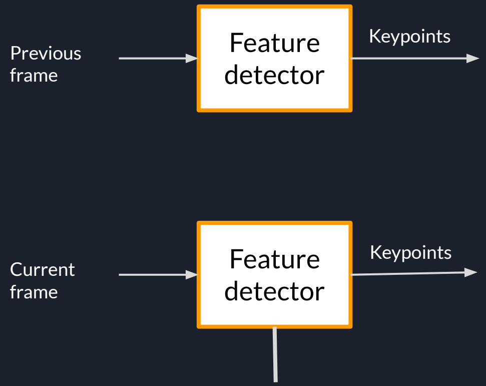
STEPS IMPLEMENTED

1. FEATURE DETECTION AND MATCHING
2. ESTIMATION OF ESSENTIAL MATRIX
3. DECOMPOSITION OF ESSENTIAL MATRIX
4. TRIANGULATION
5. LINEAR PNP (POSE FROM 3D-2D CORRESPONDENCE)
6. BUNDLE ADJUSTMENT

Project Pipeline

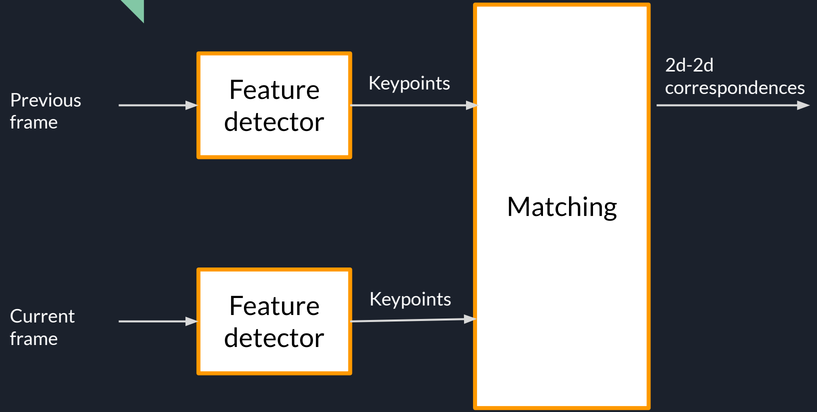


Project Pipeline

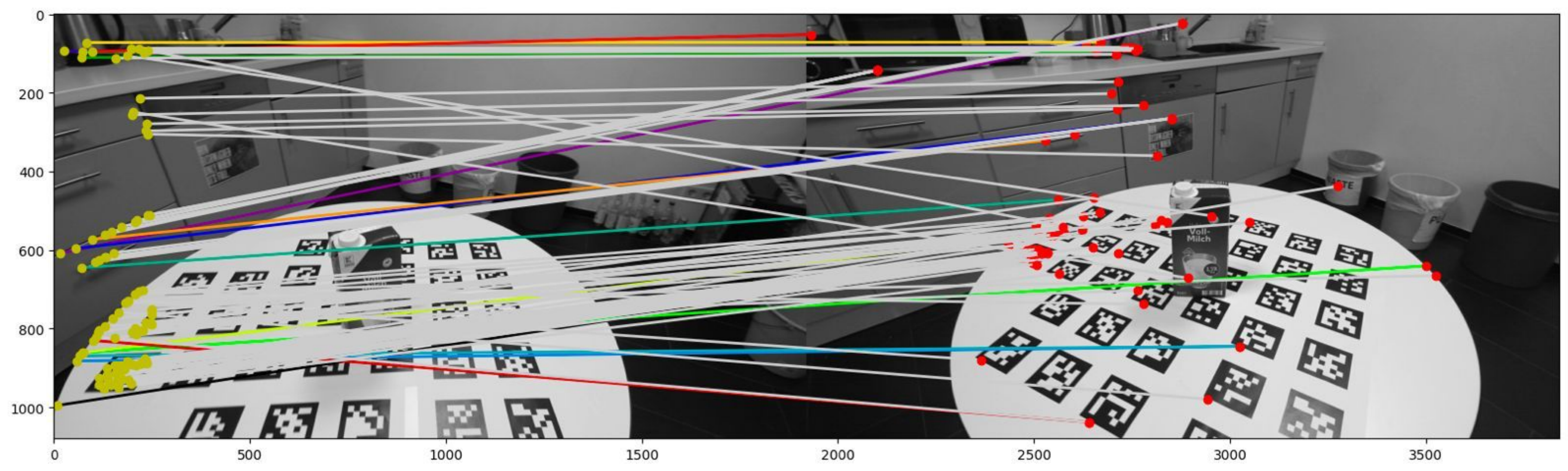


1. FEATURE DETECTION AND MATCHING

Detected Keypoints

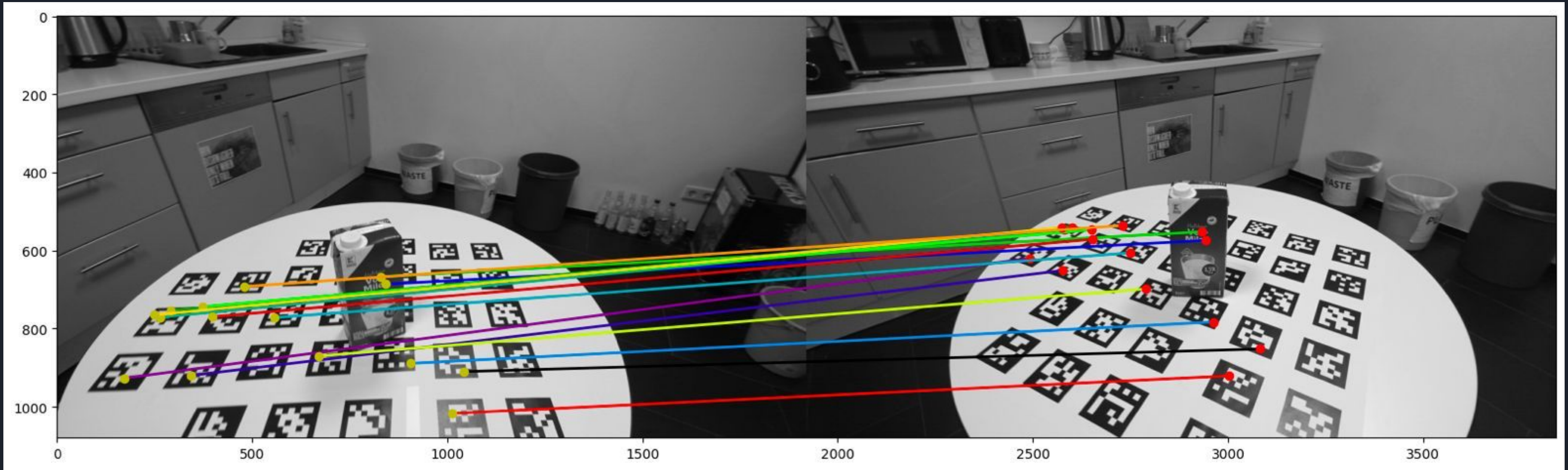

Previous
frame
Feature
detector
Keypoints
Current
frame
Feature
detector
Keypoints
Matching
2d-2d
correspondences

1. FEATURE DETECTION AND MATCHING



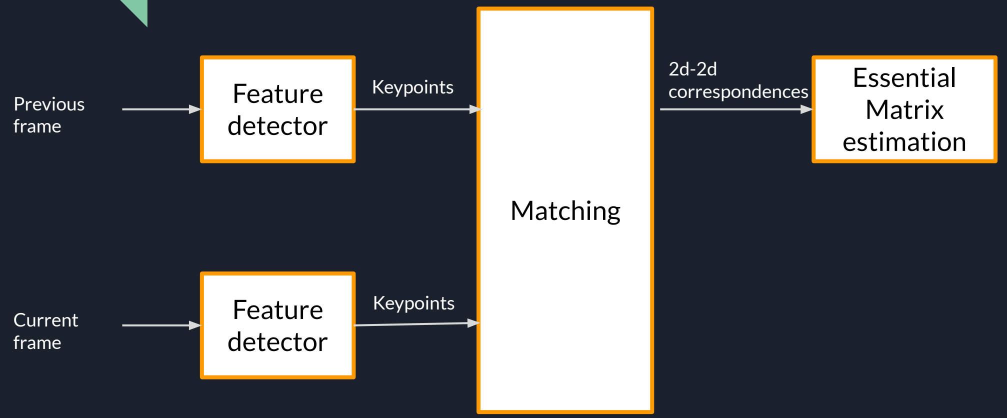
RAW MATCHES

1. FEATURE DETECTION AND MATCHING



FILTERED MATCHES

Project Pipeline



2. ESTIMATION OF ESSENTIAL MATRIX

$$x_2^T K^{-T} E K^{-1} x_1 = 0$$

EPIPOLAR CONSTRAINT

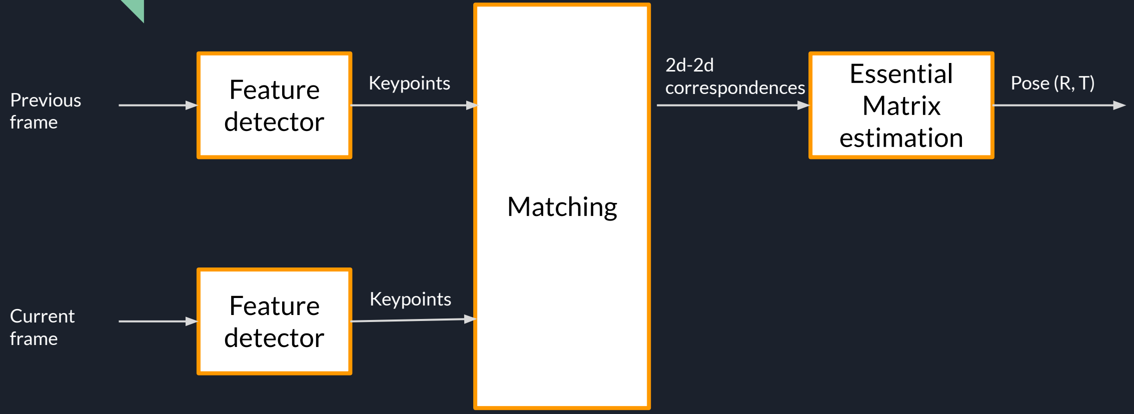
$$x_{n_2}^T E x_{n_1} = 0$$

$$\begin{bmatrix} u_i' & v_i' & 1 \end{bmatrix} \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e23 \\ e_{31} & e_{32} & e_{33} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} u_1'u_1 & u_1'v_1 & u_1' & v_1'u_1 & v_1'v_1 & v_1' & u_1 & v_1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ u_n'u_n & u_n'v_n & u_n' & v_n'u_n & v_n'v_n & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \\ e_{21} \\ e_{22} \\ e_{23} \\ e_{31} \\ e_{32} \\ e_{33} \end{bmatrix} = 0$$

- Linear least Square problem.
- **Ax = 0**
- Solved using SVD

Project Pipeline

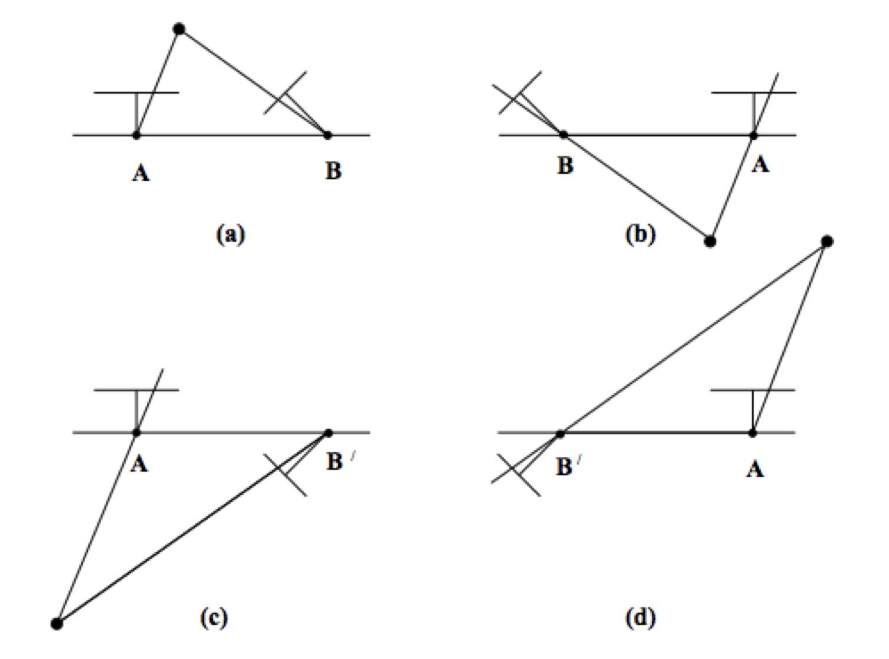


3. DECOMPOSITION OF ESSENTIAL MATRIX

$$\begin{tikzcd}
 & E = [t]_\times R \arrow[dl] \arrow[dr] & \\
t & & R
\end{tikzcd}$$

- Essential Matrix is comprised of relative Rotation and Translation

3. DECOMPOSITION OF ESSENTIAL MATRIX



Ambiguity in Pose(4 possible combination)

$$R = UWV^T \quad t = u_3$$

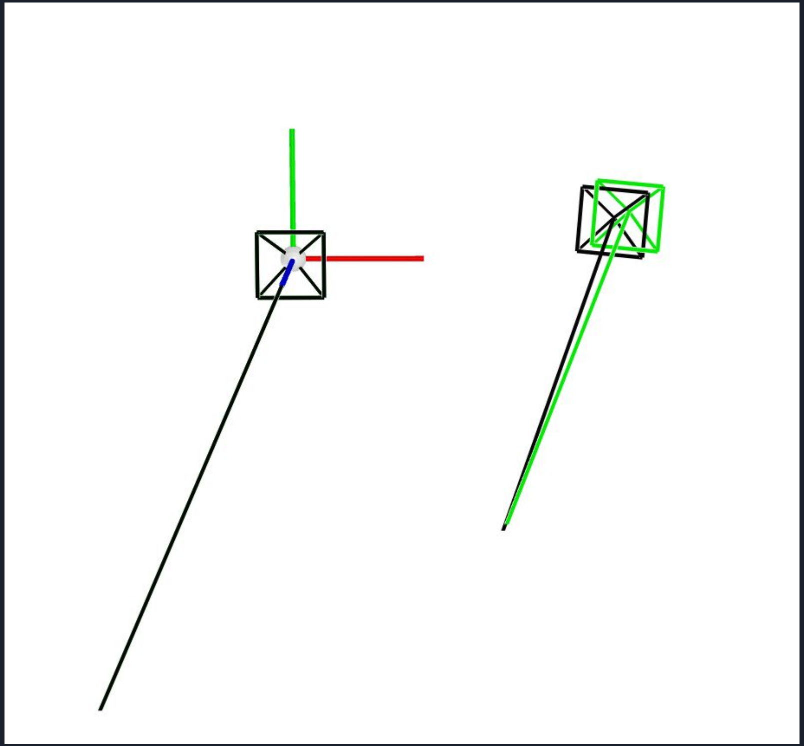
$$R = UW^TV^T \quad t = u_3$$

$$R = UWV^T \quad t = -u_3$$

$$R = UW^TV^T \quad t = -u_3$$

Triangulate to check the correct configuration

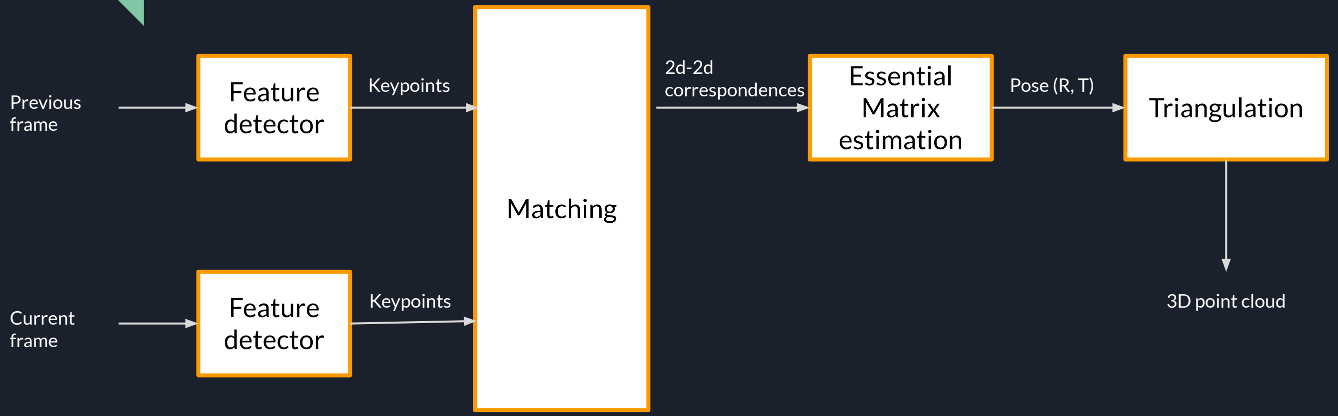
3. DECOMPOSITION OF ESSENTIAL MATRIX



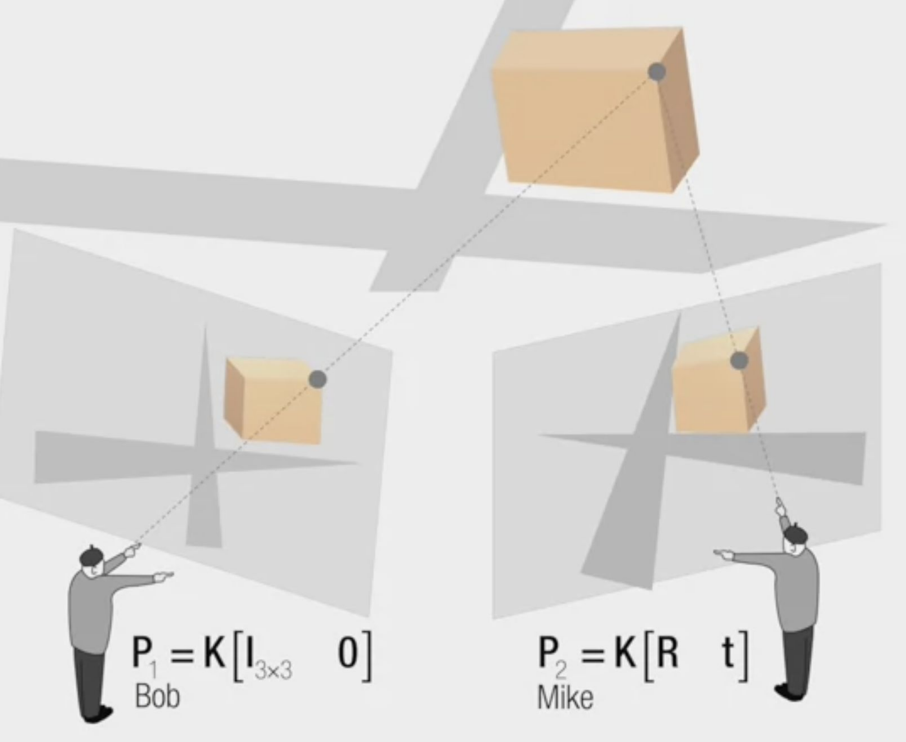
Green: Ground truth Pose Provided

Black: Estimated Pose using Essential Matrix

Project Pipeline



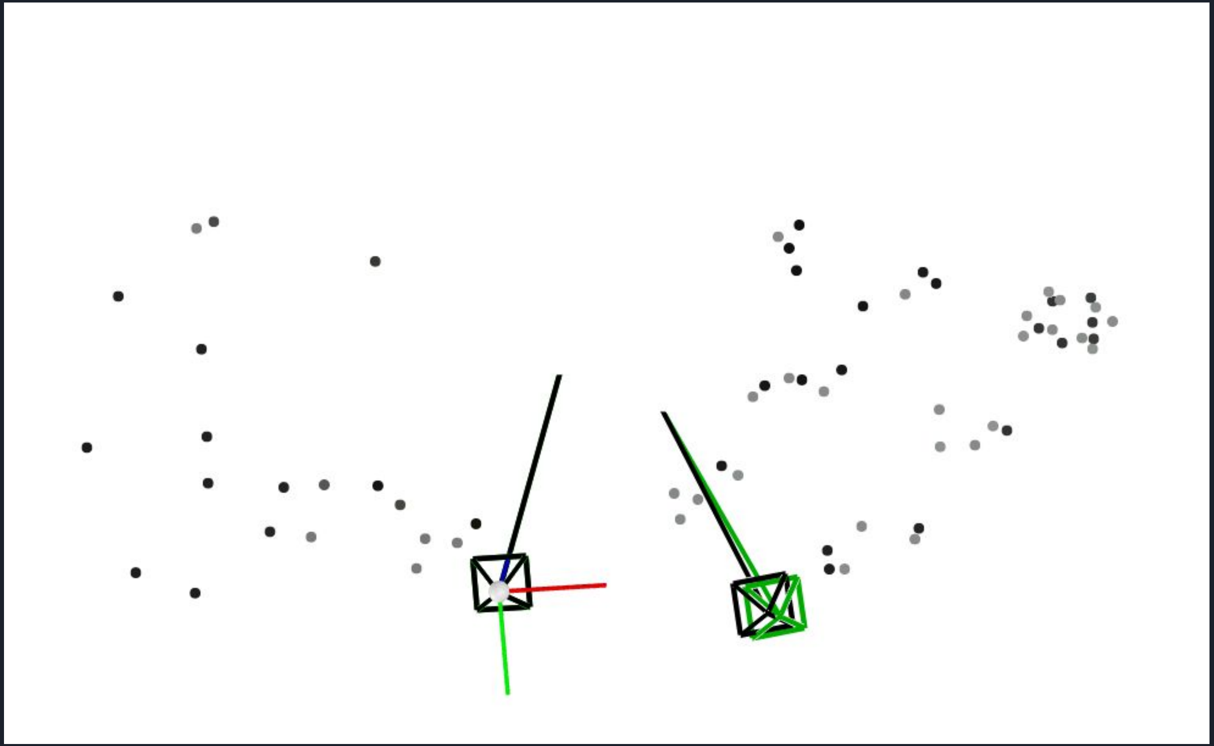
4. TRIANGULATION



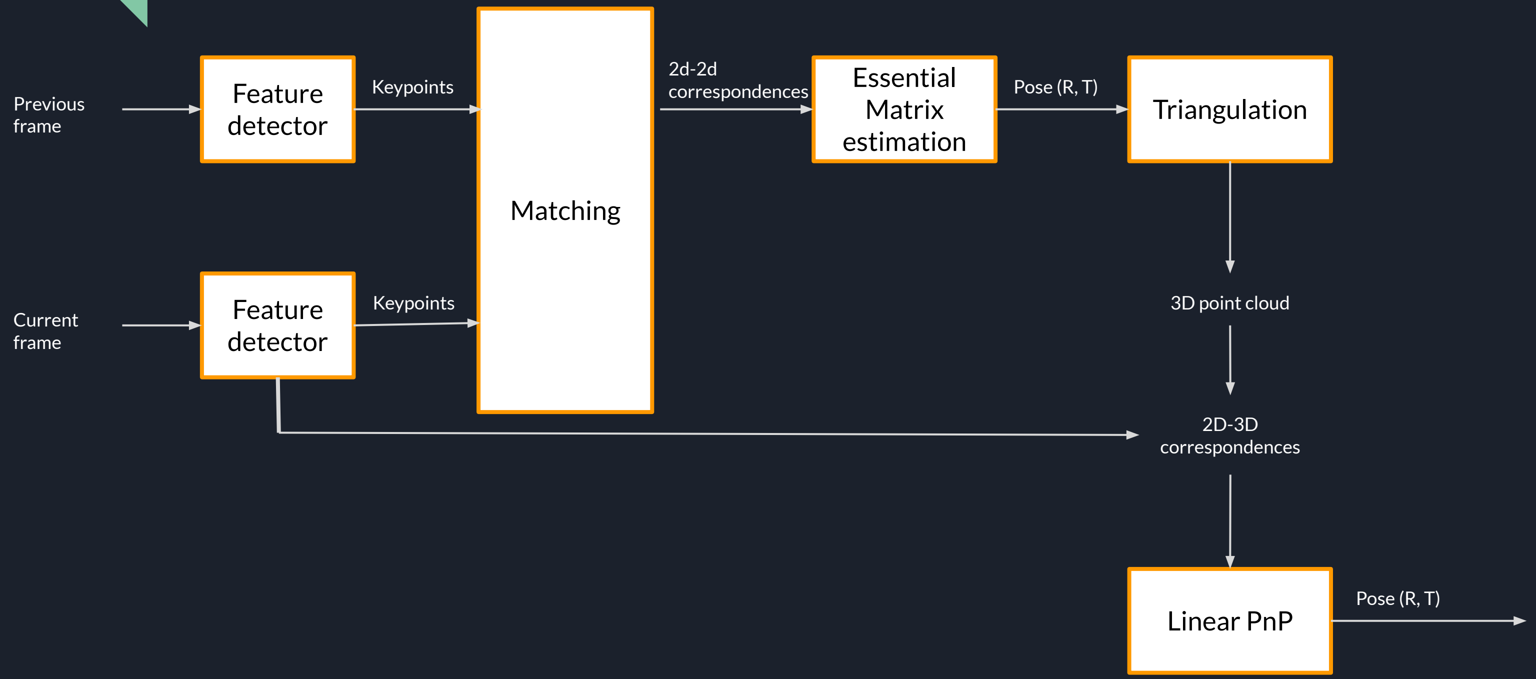
Inputs Required:
x1: Image point in Frame 1
x2: Image point in Frame 2
P1: Projection Matrix of Frame 1
P2: Projection Matrix of Frame 2

Output:
X: 3D coordinate of corresponding points in space

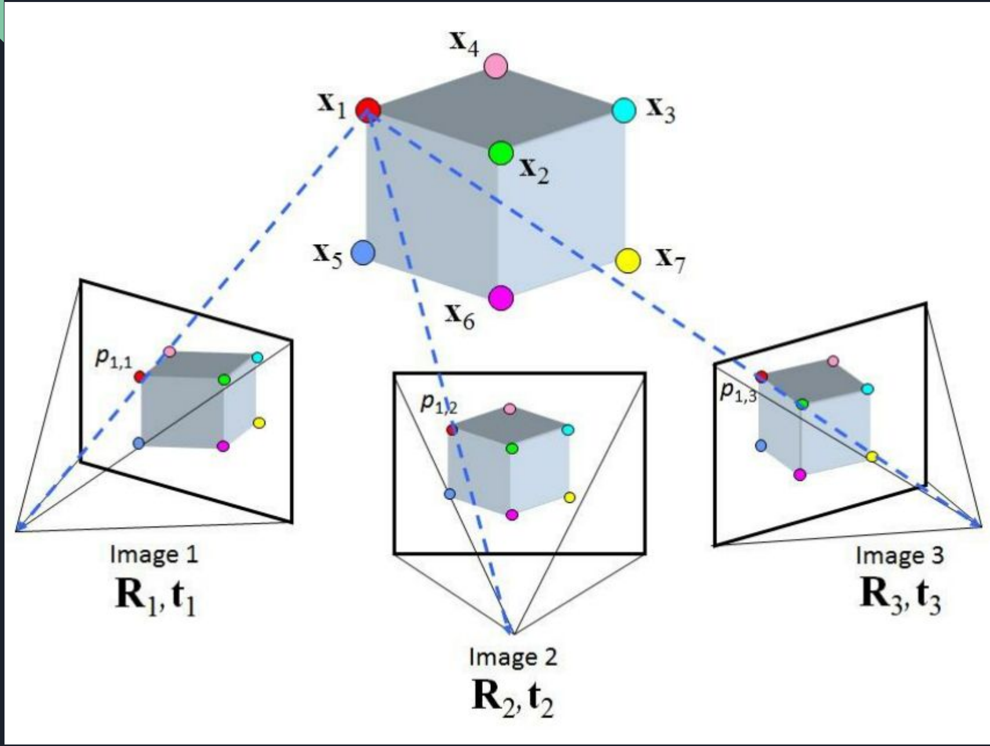
4. TRIANGULATION



Project Pipeline



5. Linear PnP



Inputs Required:
x: Image point in Frame 1
X: 3D coordinate of corresponding points in space

Output:
R, T: Pose of the Frame

5. Linear PnP

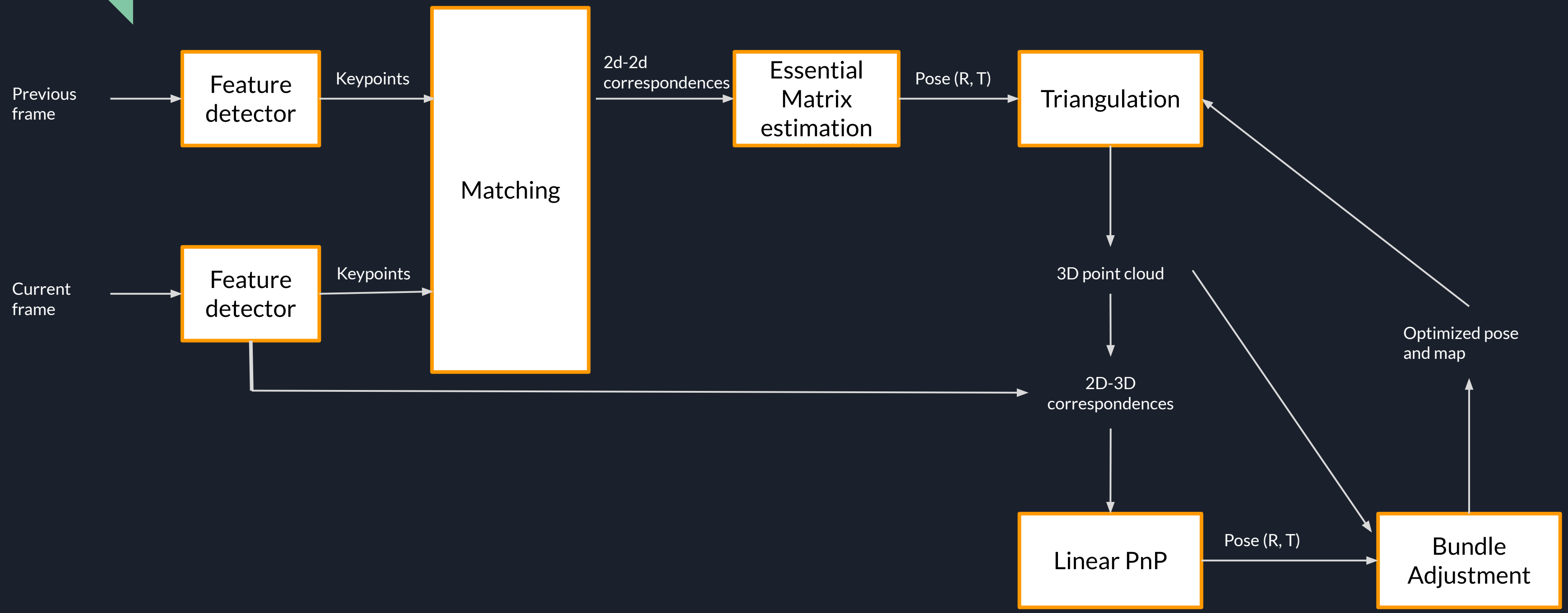
$$\begin{bmatrix} x \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} \tilde{X} = 0$$

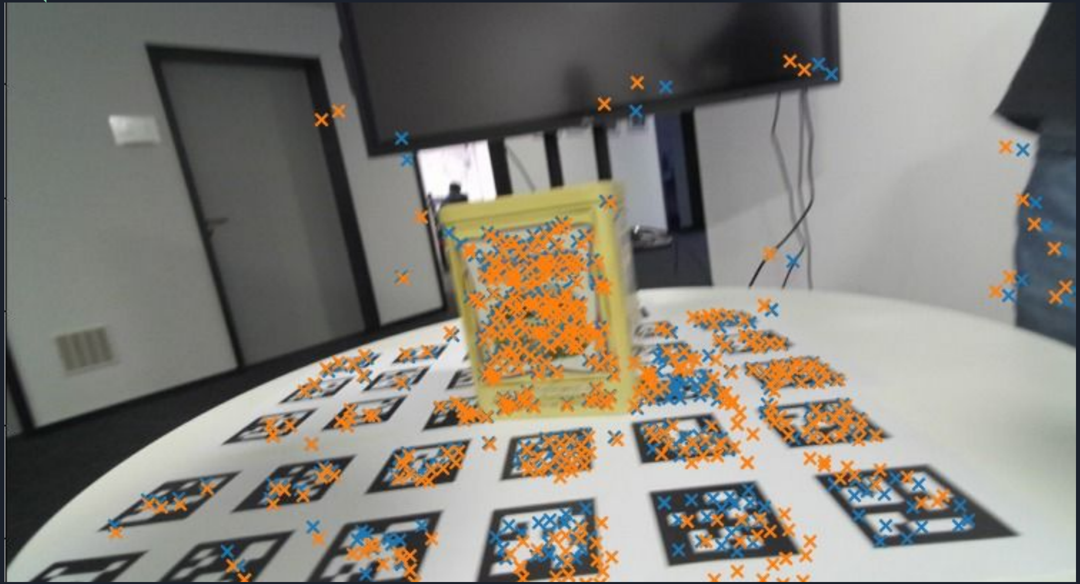
$$\begin{bmatrix} 0 & -\tilde{X}^T & v\tilde{X}^T \\ \tilde{X}^T & 0 & -u\tilde{X}^T \\ -v\tilde{X}^T & u\tilde{X}^T & 0 \end{bmatrix}_{3\times12} \begin{bmatrix} P_1^T \\ P_2^T \\ P_3^T \end{bmatrix}_{12\times1} = 0$$

- Linear least Square problem.
- **Ax = 0**
- Solved using SVD

Project Pipeline



6. Bundle Adjustment



Reprojection error

Blue: True keypoints points

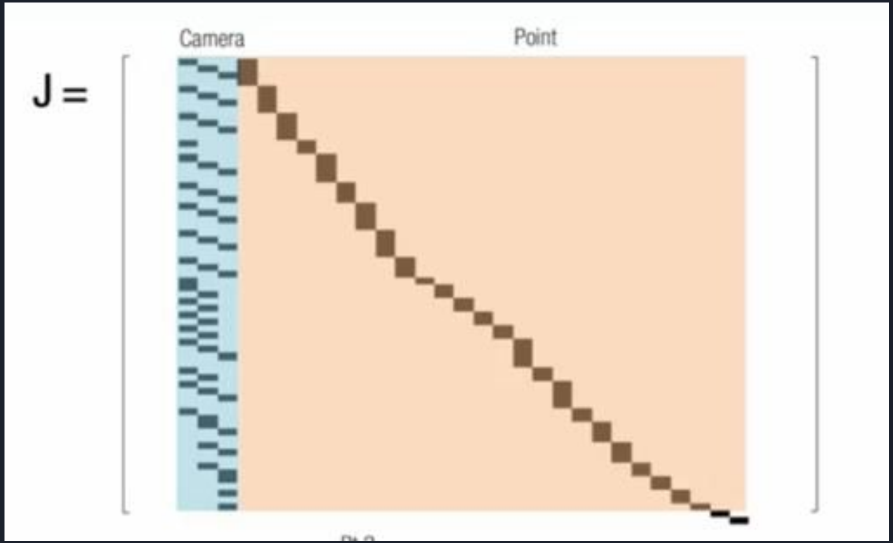
Orange: Reprojected points

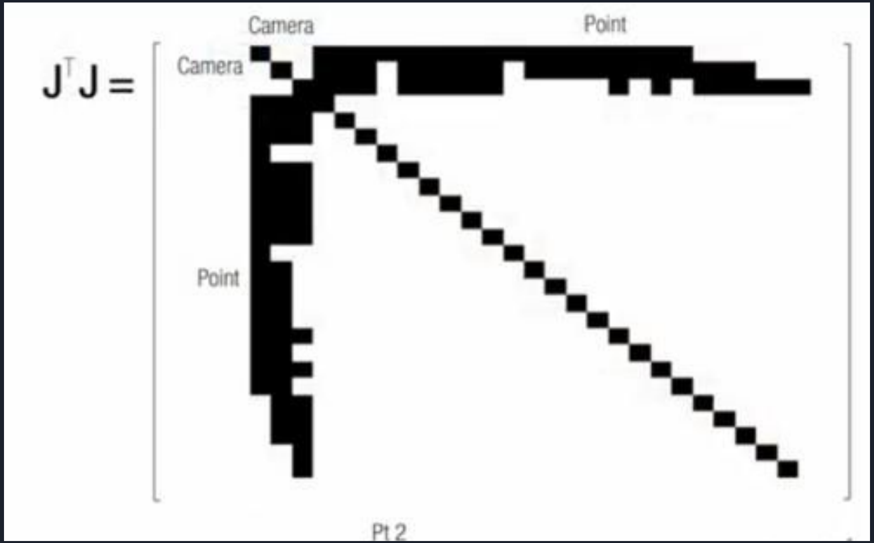
6. Bundle Adjustment

Goal: Minimize the reprojection error

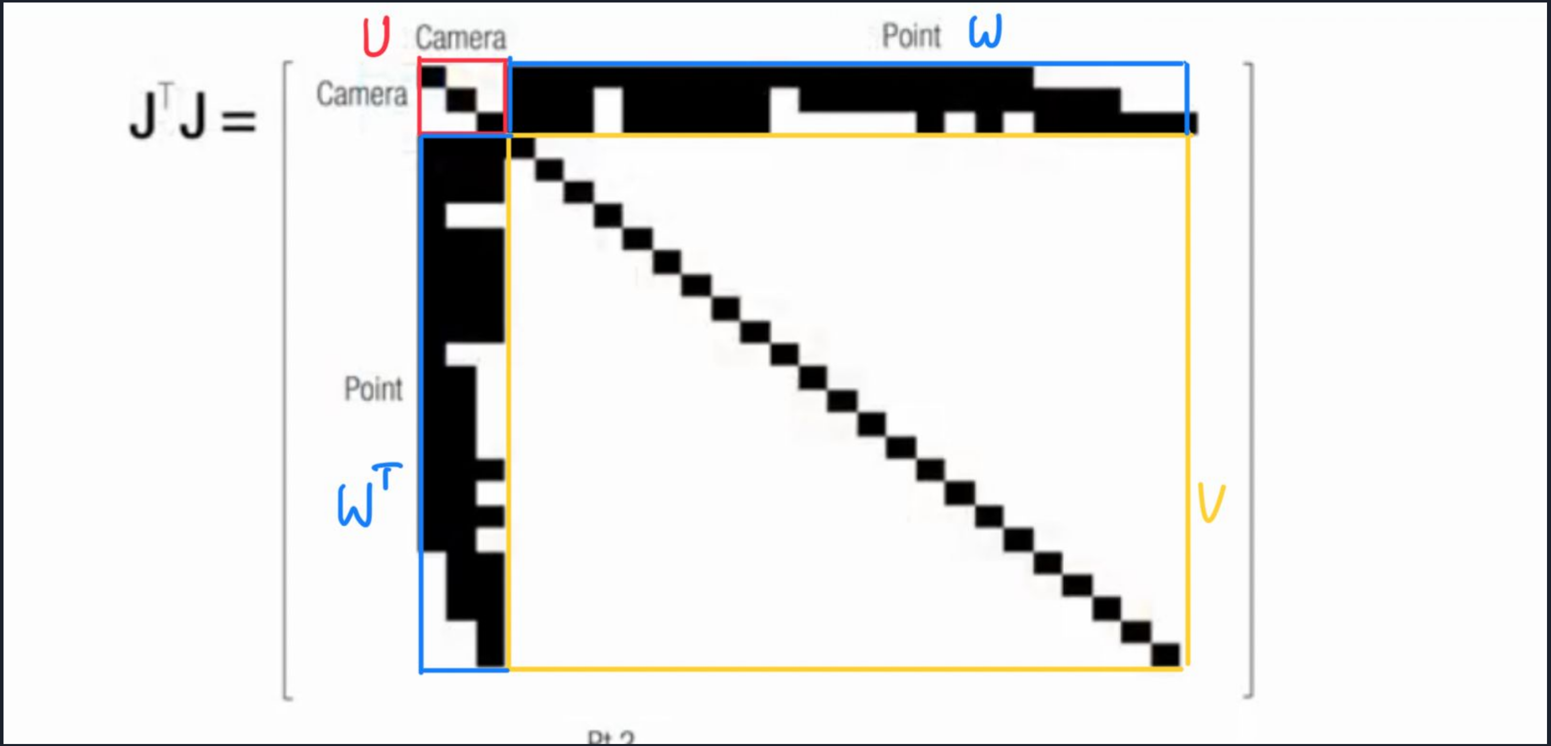
$$\Delta x = (J^T J)^{-1} J^T (b - f(x))$$

6. Bundle Adjustment





6. Bundle Adjustment



6. Bundle Adjustment

- Solve for $\Delta_\mathbf{a}$:

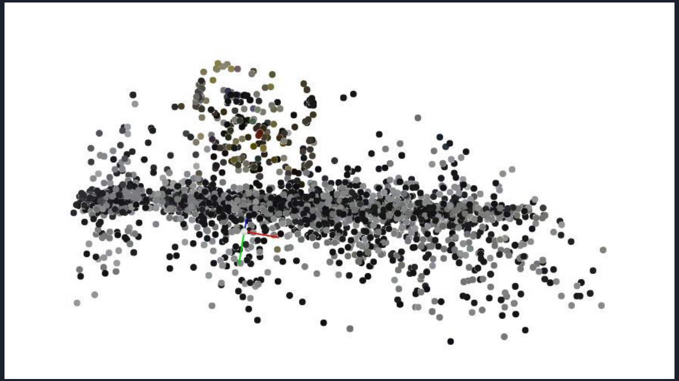
$$(\mathbf{U}^* - \mathbf{W}{\mathbf{V}^*}^{-1}\mathbf{W}^\top)\Delta_\mathbf{a} = \epsilon_\mathbf{A} - \mathbf{W}{\mathbf{V}^*}^{-1}\epsilon_\mathbf{B}$$

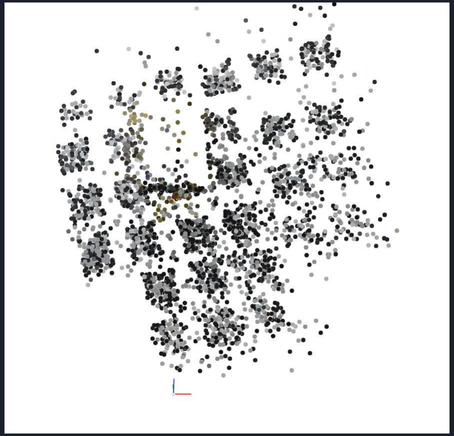
- Back-substitute for $\Delta_\mathbf{b}$:

$$\mathbf{V}^*\Delta_\mathbf{b} = \epsilon_\mathbf{B} - \mathbf{W}^\top\Delta_\mathbf{a}$$

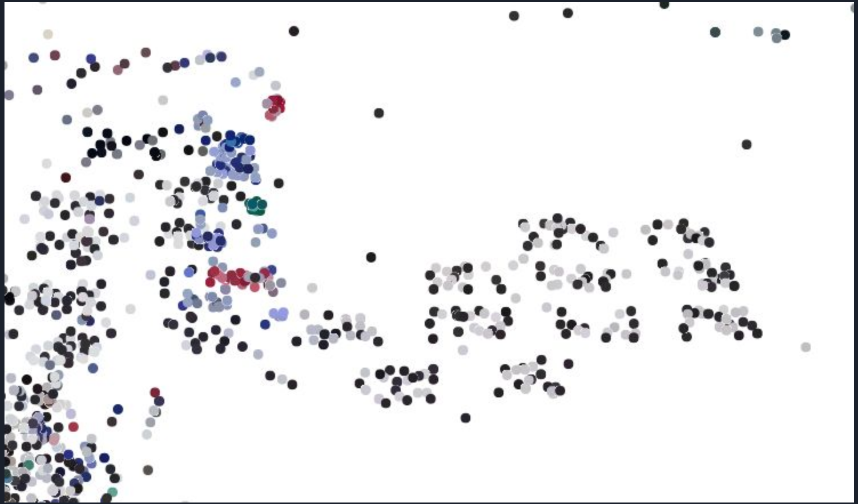
Inverse of V is easy as it is sparse diagonal matrix
Δa is camera pose updates and Δb is 3d point updates

Results using provided ground truth





Results on milk dataset



References

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- Bundle Adjustment : *Sparse Estimation in Multi-View Geometry* . University of California San Diego
- Bundle Adjustment – Part II Numerics of BA - Cyrill Stachniss
- Camera Calibration: Direct Linear Transform-Cyrill Stachniss
- Direct Solution for computing Fundamental and Essential Matrix- Cyrill Stachniss